To Be or Not To Be a Linear Equation: That Is the Question

Linear Equation in Two Variables

A linear equation in two variables is an equation that can be written in the form

$$Ax + By = C$$

where *A* and *B* are not both 0. This form is called **standard form**. In standard form the variables *x* and *y* are on the same side of the equals sign, A is positive, and any constant is on the other side of the equal sign.

A line contains an infinite number of points and each ordered pair is a solution of its corresponding equation.

To determine if an equation is linear or not, we look at the exponents. If the highest exponent on a single variable is one, then the equation is a linear equation. A single variable means that the variables are not multiplied.

Here is the standard form of a linear equation in two variables again.

$$Ax + By = C$$ Look at the x. The exponent is understood to be 1.
Look at the y. The exponent is understood to be 1.

Example 1: Let's look at several examples of linear equations, and some that are not linear equations.

$x - 3y = 2$ is a linear equation since the exponents of *x* and *y* are both one.

$y = \dfrac{2}{3}x + 3$ is a linear equation since the exponents of *x* and *y* are both one. However, the equation is not written in standard form. We will learn how to change this equation to standard form later.

$\dfrac{1}{x} + y = 9$ is not a linear equation since x is in the denominator of the fraction.

$5x + 4y = 12$ is a linear equation since the exponents of x and y are both one.

$xy = -3$ is not a linear equation since the variables *x* and *y* are being multiplied together.

$x^2 - 2x - 3 = y$ is not a linear equation since the exponent of *x* is not one.

Remember, in order for an equation to be a linear, both variables need to have an exponent of one. Usually, you will not see the exponent of one written, but we know that it is there.

An equation cannot be linear if any of the variables are in the denominator, multiplied together, squared, or under a radical.

Linear or Non-Linear

Quick Check: Now you try some examples! Determine if the following equations are linear or non-linear. Circle your answer and describe why.

$3x + 2y = 7$ linear   non-linear

$y = \dfrac{2}{3}x - 5$ linear   non-linear

$x^2 - 3x + 2 = y$ linear   non-linear

$x = -4$ linear   non-linear

$y = 2$ linear   non-linear

Graphs of Equations

A graph is a picture of the relationship between two variables, in our case, the variables x and y. The graph of a linear equation is a line.

To find a relationship between two variables we need to first find ordered pairs, or points, (x,y), that make the equation true. These are often arranged in what we call a t-table.

Example 2: If we had the equation $2x + y = 8$, we first create a t-table. Under the x we place a 3. This means to replace x by 3 and find the corresponding y-value.

We can arrange this information in a t-table.

x	y	
3	2	Gives the point (3, 2)
		The values on the left side of the chart are x values for a point.
		The values on the right side of the chart are the corresponding y-values.

Now, select other values for x or for y and find the corresponding value. Two points are sufficient to draw a line, but we usually ask for three. The third point allows us to check the accuracy of our answers.

Now sketch a graph of this equation, $2x + y = 8$, on you graph paper.

Intercepts

We have already talked about graphing equations using ordered pairs, but we can also graph linear equations by using the *x*-intercept and the *y*-intercept. The x and y intercepts are ordered pairs.

> The x-intercept is the point(s) where a graph crosses the x-axis
> The y-intercept is the point(s) where a graph crosses the y-axis

To find the x-intercept, let y be zero and find x.
To find the y-intercept, let x be zero and find y.

Example 3: Let's look at the equation, $2x + 3y = 6$. To graph by intercepts:

x	y	
3	0	Replace y by 0 and find x.
0	**2**	Replace x by 0 and find y.

When we graph a linear equation using intercepts, we use only the two points. If the x- and y-intercept is $(0, 0)$, we find another point or two.

Here are the steps:

x-intercept   y-intercept

$$2x + 3y = 6$$
$$2x + 3(0) = 6$$
$$2x + 0 = 6$$
$$\frac{2x}{2} = \frac{6}{2}$$
$$x = 3$$

$$2x + 3y = 6$$
$$2(0) + 3y = 6$$
$$0 + 3y = 6$$
$$\frac{3y}{3} = \frac{6}{3}$$
$$y = 2$$

So the x-intercept is (3, 0) and the y-intercept is (0, 2).

Example 4:

Now you try to find the intercepts for the equation $3x - 4y = 12$.

a. Start by finding the x-intercept   b. Now, find the y-intercept   c. Create a t-table

$$3x - 4y = 12$$
replace y by 0

$$3x - 4y = 12$$
replace x by 0

To Graph an Equation

To graph equations in two variables:

1) Plot points that make the equation true. Find these points by **choosing** a value for either x or y and then solving for the value of the other variable.

2) Then draw a line (also called a curve) through the points. It is like the dot-to-dot pictures you may have done as a child.

We will concentrate on linear equations in this material. However, the process is the same no matter the form of the equation your given.

Remember that the graph of every linear equation in two variables is a line. Only two points are required to graph a linear equation.

Graphing Examples

In these examples we want to determine whether each equation is linear or not and then graph the equation.

Here are some suggestions: Find the x- and y-intercepts, if possible.
Plot the x-intercept and the y-intercept on the coordinate plane.
Draw a line (or curve) through the points. (Play connect-the-dots.)

Example 5:
$$y - x = 4$$

Is the equation linear or non-linear?

The equation is linear since both the exponents of x and y are one.

Now find the *x*-intercept and *y*-intercept.

To find the *x*-intercept, set $y = 0$.

$$(0) - x = 4$$
$$-x = 4$$
$$\frac{-x}{-1} = \frac{4}{-1}$$
$$x = -4$$

To find the *y*-intercept, set $x = 0$

$$y - (0) = 4$$
$$y = 4$$

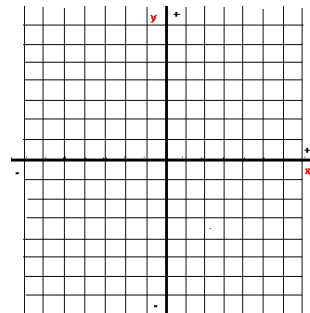x-intercept __(-4,0)__   y-intercept __(0, 4)__
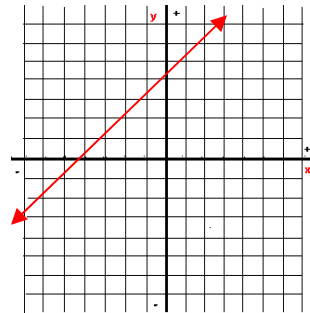
Now, you try several. Determine whether each equation is linear or not and then graph the equation.

Example 6:
$$y = 6x - 5$$

Is the equation linear or non-linear? Why?

Now, find the *x*-intercept and *y*-intercept.
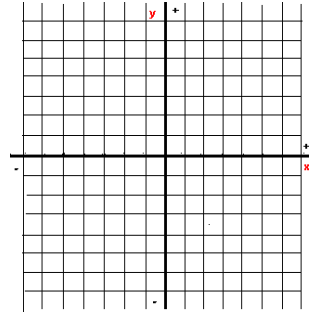
x-intercept _____   y-intercept _____

Example 7:

$$y = 3$$

Is the equation linear or non-linear? Why?

Now, find the *x*-intercept and *y*-intercept.

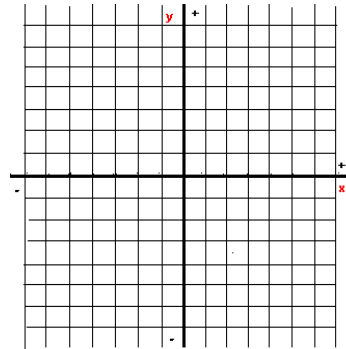x-intercept _____   y-intercept _____

If the x -and y-intercepts are the same, find two more points. Select values for x and find y or select values for y and find x.

Example 8:

$$y = \frac{2}{3}x$$

Is the equation linear or non-linear? Why?

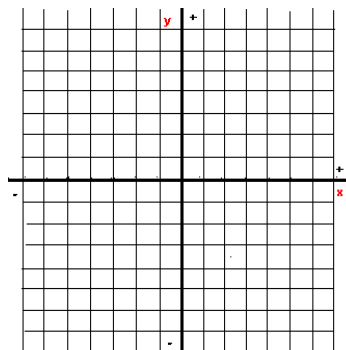Now, find the *x*-intercept and *y*-intercept.

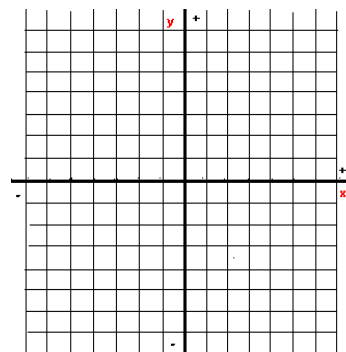x-intercept _____   y-intercept _____

Example 9:

Here is an interesting problem. On the same set of axes, graph $y = 2x$, $y = x$, and, $y = -2x$.
Create a table for each equation, to organize your points. (This activity is a good place to use those map colors you never used in high school.)
Describe the differences and similarities in these graphs.

Example 10:

And here is another interesting problem. On the same set of axes, graph $y = 2x$, $y = 2x + 3$, and $y = 2x - 4$. Describe the differences and similarities in these graphs.

In summary, a linear equation in two variables looks like $Ax + By = C$ in standard form. An equation is only linear if the variables have exponents of one. We can graph linear equations by plotting points or using the x-intercept and y-intercept. Remember that we only need two points to graph the equation of a line. If the x-intercept and y-intercept are the same, then we need to find additional ordered pairs to help us graph the linear equation.