

Equations of Lines: The Finale

Now for the big finish!! Let's learn how to write an equation of a line. So far we have talked about two forms of an equation of a line. Standard form looks like $Ax + By = C$, where the coefficient of x is positive and there are no fractions. This form isn't very useful to graph, but most equations are given to us in this form. Another form of an equation of a line is slope-intercept form. Remember that slope intercept form looks like $y = mx + b$ or in function notation, $f(x) = mx + b$. In this form of an equation of a line, m represents the slope and b represents the y -intercept, $(0, b)$.

To write the equation of a line in slope-intercept form we need two pieces of information:

- 1) the slope
- 2) the y -intercept

Examples: Write the equation of the line in slope-intercepts form, given the following pieces of information.

1. We are given the following pieces of information:
- 1) $m = -\frac{2}{3}$
 - 2) $b = 5$

Since we know the slope and the y -intercept, it is appropriate to use the slope-intercept form of the equation:

$$y = mx + b \quad \text{replace } m \text{ with } -\frac{2}{3}$$

$$y = -\frac{2}{3}x + b \quad \text{replace } b \text{ with } 5$$

$$y = -\frac{2}{3}x + 5 \quad \text{This is the slope-intercept equation for the line.}$$

2. We are given the following pieces of information:
- 1) $m = \frac{3}{8}$
 - 2) point $(0, -3)$

We need to recognize that $(0, -3)$ is the y -intercept and that $b = -3$.

$$y = mx + b \quad \text{replace } m \text{ with } \frac{3}{8}$$

$$y = \frac{3}{8}x + b \quad \text{replace } b \text{ with } -3$$

$$y = \frac{3}{8}x - 3 \quad \text{This is the slope-intercept equation for the line.}$$

Now, you try some examples. Use the information that is given to find the equation of the line in slope-intercept form.

3. We are given the following pieces of information:
- 1) Slope 2
 - 2) y-intercept $\left(0, \frac{2}{3}\right)$

4. We are given the following pieces of information:
- 1) $m = -4$
 - 2) y-intercept (0,1)

Graphing the equation of the line in slope-intercept form

To graph a line using slope intercept form, first graph the y-intercept. Then, to find another point on the line, use the slope (rise/run) from the y-intercept.

Examples: Graph the following equations of lines using the slope and y-intercept.

5. $y = 2x + 3$

The slope is 2.

The y-intercept is 3.

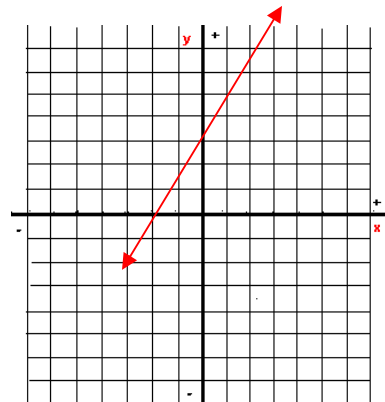
We start at 3 on the y-axis since this is the y-intercept.

Then use the slope: $2 = \frac{2}{1} = \frac{+2 \text{ change in } y}{+1 \text{ change in } x}$

from 3 on the y-axis, move up 2 units and then right 1 unit.

Place a point.

Finally, draw a line through the two points.



Now, here is one where we have to solve our given equation for y in order to get our equation in slope-intercept form.

6. $-3x + 2y = 3$

Solve for y .

$$-3x + 2y = 3$$

$$+3x \quad +3x$$

$$2y = 3x + 3$$

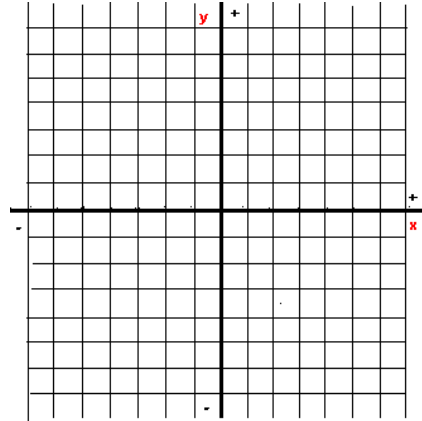
$$\frac{2y}{2} = \frac{3x}{2} + \frac{3}{2}$$

$$y = \frac{3}{2}x + \frac{3}{2}$$

$$m = \frac{3}{2} \quad b = \frac{3}{2}$$

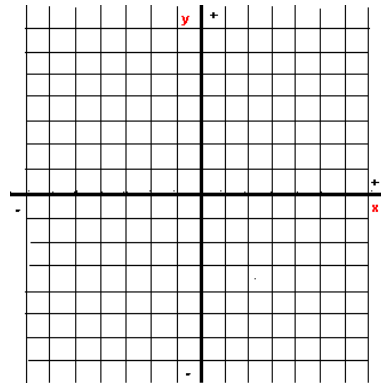
Start at $\frac{3}{2}$ on the y -axis.

Then, to find the second point, count up 3 from the point $\frac{3}{2}$ on the y -axis and then right 2 to places.



Okay, now you try this one. Graph the equation of the line using slope and the y-intercept.

7. $y = 5x$



Another useful form of an equation of a line is called point-slope form. We like to use this equation when we are given at least one point and the slope of the line.

Point-Slope Form of the Equation of a Line: $y - y_1 = m(x - x_1)$

The point-slope form of the equation of a line is $y - y_1 = m(x - x_1)$, where m is the slope of the line and (x_1, y_1) is a point on the line.

The two bits of information given are the slope, m , and the point, (x_1, y_1) .

Some students remember the formula for the slope of a line to help remember the point-slope form of the equation of a line.

Remember that the equation to find the slope of a line is:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

For this problem, however, you have the slope, m , and only one point (x_1, y_1)

In the slope formula replace m and the point, (x_1, y_1) .

$$m = \frac{y - y_1}{x - x_1}$$

Since there is no second point, (x_2, y_2) , those subscripts are not used.

$$(x - x_1)m = \frac{y - y_1}{x - x_1}(x - x_1)$$

Multiply by $(x - x_1)$ to clear the fraction.

$$(x - x_1)m = y - y_1$$

Results in the point-slope form of the line.

$$y - y_1 = m(x - x_1)$$

Let's practice finding the equation of each line with the given conditions, using point-slope form.

Examples: Find the equation of the line, in slope-intercept form, with the given conditions.

8. Given: $m = \frac{1}{3}$ and a point $(6, -2)$

Note that $x_1 = \underline{6}$ and $y_1 = \underline{-2}$

To write the equation of a line with the slope and a point given, place the values into the corresponding position in the formula.

Begin by writing the formula:

$$y - y_1 = m(x - x_1)$$

$$y - y_1 = \frac{1}{3}(x - x_1) \quad \text{replace } m \text{ with } \frac{1}{3}$$

$$y - y_1 = \frac{1}{3}(x - 6) \quad \text{replace } x_1$$

$$y - (-2) = \frac{1}{3}(x - 6) \quad \text{replace } y_1$$

$$y + 2 = \frac{1}{3}x - 2 \quad \text{distribute } \frac{1}{3}$$

$$y = \frac{1}{3}x - 4 \quad \text{Solve for } y!$$

Now you try some examples. Find the equation of the line, in slope-intercept form, with the given conditions.

9. Find the equation of the line with slope 3 and goes through the point (-1, 2).

Think: $m =$ _____, $x_1 =$ _____, and $y_1 =$ _____

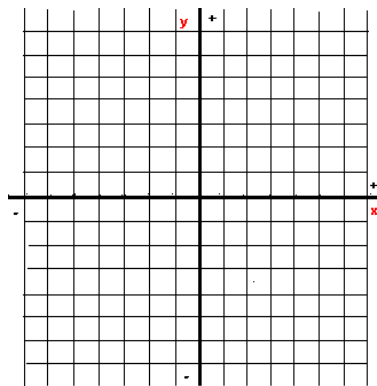
Write the point-slope form:

$$y - y_1 = m(x - x_1)$$

Replace the know values for the variables.
Solve for y.

The equation you found is _____.

Graph this equation.



10. Find the equation of the line with slope $-\frac{3}{5}$ and goes through the point (4, -1).

Think: $m = \underline{\hspace{2cm}}$, $x_1 = \underline{\hspace{2cm}}$, and $y_1 = \underline{\hspace{2cm}}$

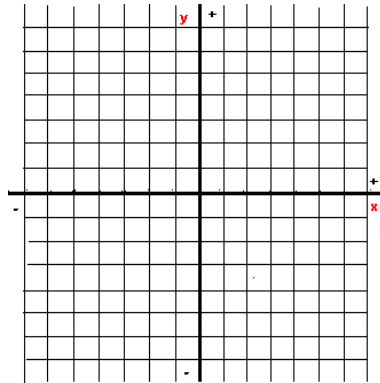
Write the point-slope form:

$$y - y_1 = m(x - x_1)$$

Replace the know values for the variables.
Solve for y.

The equation you found is _____.

Graph this equation.



What happens when we are given two points and no slope? We can still use the point-slope form of the equation of a line, but we need to use the two points to find the slope first.

Examples: Find the equation of the line, in slope-intercept form, with the given conditions.

Let's do one together first.

11. Find the equation of the line that goes through the points $(-7,-4)$ and $(0,-6)$.

Since you have two points, find the slope first.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{-6 - (-4)}{9 - (-7)}$$

$$m = \frac{-2}{16}$$

$$m = \frac{-1}{8}$$

Now, look at the two points we were given. Notice that $(0,-6)$ is the y-intercept. So, we can use slope-intercept form to find the equation of the line.

$$y = mx + b$$

$$y = -\frac{1}{8}x - 6$$

If you do not notice that $(0,-6)$ is the y-intercept, we can use point-slope form to find the equation of the line. You may use either point for (x_1, y_1) and the m you found.

$$y - y_1 = m(x - x_1)$$

I am going to use the point (0, -6) and the slope found above, $m = \frac{-1}{8}$.

$$y - (-6) = \frac{-1}{8}(x - (0))$$

$$y + 6 = \frac{-1}{8}x - 0$$

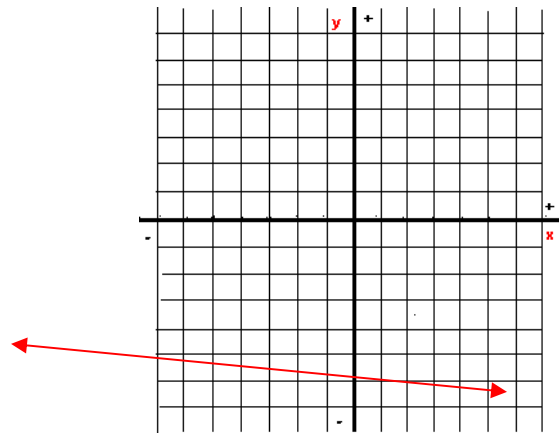
$$y + 6 = \frac{-1}{8}x$$

$$\begin{array}{r} -6 \qquad \qquad -6 \\ \hline \end{array}$$

$$y = \frac{-1}{8}x - 6$$

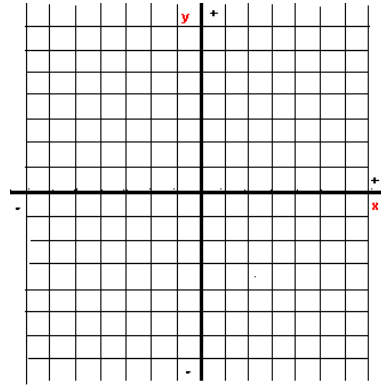
So, the equation of the line between the points (-7, -4) and (0, -6) is $y = \frac{-1}{8}x - 6$. We could also write the equation of the same line using function notation. The equation of the line between the points (-7, -4) and (0, -6) is $f(x) = \frac{-1}{8}x - 6$.

Graphing the equation looks like:



Now, you try this one.

12. Find the equation of the line that goes through the points $(-4,3)$ and $(2,-5)$. Then graph the line.



In summary, there are three common forms of an equation of a line. The three forms include:

- 1). Standard Form $Ax + By = C$
- 2). Slope-Intercept Form $y = mx + b$ or $f(x) = mx + b$
- 3) Point-Slope Form $y - y_1 = m(x - x_1)$

Depending on the information we are given, it is more appropriate to use one form over the other. With practice, it becomes easier to identify which form is more appropriate.